

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS  
METHODS  
UNIT 3

**SOLUTIONS**

Section One:  
Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (52 Marks)

This section has**eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

(a) Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of chain rule  ü correct derivative |

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* Generally ok. Main error was forgetting to use the chain rule and the negative sign.

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(b) Evaluate when . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of quotient rule  ü correct and  ü correct derivative  ü substitutes and simplifies |

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* Finding the derivative by using the quotient rule was done well by the majority.
* Main error was incorrect exact values. Many had cos( rather than -1. Students must know their exact values for both sin and cos.

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Question 2 (5 marks)

A small body is initially at the origin. It is moving along the -axis with velocity at time seconds given by

(a) Determine , a function for the displacement of the body at time . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ reasonable attempt at using chain rule  ü correct antiderivative  ü correct displacement function |

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* Finding the integral was ok but not brilliant. Many still need to work on their basics.
* Main error was assuming +c = 0 rather than substituting and evaluating for c.

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The small body is stationary when .

(b) Determine the displacement of the body at seconds. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value of  ü correct displacement |

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* Some did not recognise that v(t)= 0 is when the body is stationary. Algebra was also lacking in solving a cubic equal to 0.
* Many substituted T+3 into the velocity equation rather than finding the value of T and then adding 3.

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Question 3 (6 marks)

Determine the area of the finite region bounded by and .

|  |
| --- |
| **Solution** |
| Points of intersection:  Area: |
| **Specific behaviours** |
| ✓ equates curves and squares  ü points of intersection  ü writes integral for area  ü correct antiderivative  ü substitutes  ü simplifies to obtain area |

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* Poorly done question. Students must be able to find the area between curves. More work required on this topic.
* Major errors include not able to solve two equations that are equal. Very poor algebra. Squaring not More work required on basic algebraic skills.
* Most understood the required integration and had the correct integral statement. Integrating caused many problems. Again basic integral skills need improving. Many students had rather than .

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Question 4 (8 marks)

(a) State three key characteristics of a chance experiment that make it suitable for modelling by a binomial random variable. (3 marks)

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| --- |
| **Solution** |
| 1. There are a fixed number of identical and independent trials.  2. There are only two possible outcomes for each trial ('success' and 'failure').  3. The probability of 'success' is the same in every trial. |
| **Specific behaviours** |
| ✓ identifies one characteristic  ✓ identifies second characteristic  ✓ identifies third characteristic |

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* Quite generous with many comments. If there are 3 marks then find 3 comments – this was also stated in the question.
* Many forgot *independent trials* and *two possible outcomes for each trial.*
* In future start *Multiple Bernoulli trials* rather than just *multiple trials*.

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Research has shown that of dogs between the ages of and have some form of heart disease. A random sample of dogs is selected from a large number of dogs of this age. Let be the number of dogs in the sample with some form of heart disease.

(b) Explain why randomly selecting one dog and recording whether it has some form of heart disease is a Bernoulli trial. (1 mark)

|  |
| --- |
| **Solution** |
| It is a **chance experiment** (dog is selected at random) with **two possible outcomes** (dog has some form of heart disease, or it does not). |
| **Specific behaviours** |
| ✓ mentions both bolded phrases, or their equivalent |

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* Generally well done.

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(c) Write a numerical expression for the probability that dogs in the sample have some form of heart disease. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü indicates binomial distribution  ✓ correct expression |

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* Ok but not brilliant.
* State that the distribution is binomial and its associated parameters.
* Include the values in the binomial distribution formula found on the formula sheet.

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(d) State the mean and variance of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct mean  ü correct variance |

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* Ok but not brilliant. Many did not use n = 70 and used only 0.1 and 0.9.More work needed on expected value and variance.

Question 5 (7 marks)

A four-sided die has faces marked with the numbers and . All faces have an equal chance of landing face down after the die is rolled. A game, that costs to play, involves throwing the die twice and adding the two numbers that land face down. If the total score is , the player wins , and otherwise they win nothing.

Let be the total score obtained in one play of the game.

(a) Construct a probability distribution table for . (3 marks)

|  |
| --- |
| **Solution** |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |
| **Specific behaviours** |
| ✓ table with label and correct values  ü label and at least two correct probabilities  ü wholly correct pd table |

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* Generally well done.
* Major errors were not adding the two values together and some incorrect probabilities.

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(b) Determine . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü correct |

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* Well done.

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Let be the net monetary loss, in dollars, of a player in **two** plays of the game.

(c) Determine . (3 marks)

|  |
| --- |
| **Solution** |
| |  |  |  | | --- | --- | --- | |  |  |  | |  |  |  |   Let be monetary loss in one game, then .  Hence . |
| **Specific behaviours** |
| ✓ indicates possible losses with probabilities in one game  ü indicates expected loss in one game  ü calculates |

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* Not well done by the majority.
* Most did not read the question – two plays and Y = monetary loss.
* Incorrect values for t and P(T = t).

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Question 6 (5 marks)

(a) Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses product rule  ü obtains correct result |

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* Generally well done.

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(b) Hence, or otherwise, determine . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ integrates all terms of result from (a)  ü uses fundamental theorem to simplify LHS  ü obtains required result, with constant |

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* Poorly done by most students. More work needed on this topic as this is a common question.
* First statement was often done well. Poor setting out resulted in incorrect working. On LHS - Integral of derivative resulted in . Many then took the integral of this rather than using the RHS integral of .

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Question 7 (8 marks)

The function is defined by , so that .

(a) Describe the concavity of the graph of . (4 marks)

|  |
| --- |
| **Solution** |
| is concave up when and .  is concave down when . |
| **Specific behaviours** |
| ✓ solves  ü indicates sign of in three intervals  ü states domains for concave up, down  ü uses correct inequalities in domains  *(penalise ambiguous language such as between and , etc.)* |

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* Badly done question. Most had no idea of concavity and its relationship with . Many played around with .
* Most commented on concavity related to rather than on the values of x.
* This needs much more work.

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(b) Determine, with justification, the range of . (4 marks)

|  |
| --- |
| **Solution** |
| As .  Minimum and maximum of will be when its derivative , (i.e., at points of inflection) and from part (a) this is when .  Hence the range is: |
| **Specific behaviours** |
| ✓ expression for  ü states behaviour of for  ü location of minimum and maximum values of  ü correct range, as inequality |

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* Many blank answers. First derivative found in most cases correctly. Most had no idea of how to find the range of values.
* Poorly done and again needs more work.

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Question 8 (7 marks)

The following table shows the probability distribution for the random variable .

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

(a) Determine the value of the positive constant and hence state . (4 marks)

|  |
| --- |
| **Solution** |
| Hence |
| **Specific behaviours** |
| ✓ sums probabilities to  ü forms quadratic equal to  ü solves quadratic, states unique value of  ü states probability |

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* The majority of students knew that the probabilities add to 1.
* Poor algebra skills in solving this algebraic fraction = 1 resulted in many not completing this question. Not knowing how to solve the equation or rewriting created problems. This needs urgent attention.
* Poorly done.

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The random variable .

(b) Determine and . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  ü indicates  ü |

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* As a result of many not completing part(a) this was often not done.
* Those who completed part(a) correctly did this well.

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Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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